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Deconstructing de Sitter

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Abstract: Semiclassical gravity predicts that de Sitter space has a finite entropy. We suggest a picture for Euclidean de Sitter space in string theory, and use the AdS/CFT correspondence to argue that de Sitter entropy can be understood as the number of degrees of freedom in a quantum mechanical dual.

Keywords: M-Theory, AdS-CFT Correspondance.

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Contents

1. Introduction

Current astronomical observations suggest that the universe is well-approximated by a world with a positive cosmological constant $\Lambda > 0$ [1]. If this is the case, string theorists must contend with two strange facts. First, a small, positive cosmological constant is surprisingly hard to construct within the usual framework of string theory and the low energy supergravities that arise from it. Of course, we might expect that nothing prevents the generation of a cosmological constant after supersymmetry is broken. However, finding a satisfactory SUSY breaking mechanism is itself a difficult problem in string theory. We might suppose that this state of affairs is simply a failure of imagination among theorists, but there is a more troubling fact. De Sitter space is expanding so rapidly that inertial observers see a cosmological horizon, and application of the Bekenstein-Hawking bound to this horizon implies that D dimensional de Sitter space has an entropy $S \sim \Lambda^{-(D-2)/2}/G_N$. This is particularly disturbing since it suggests that the entire observable universe should perhaps have a finite number of degrees of freedom. This interpretation would certainly contradict both field theory and perturbative string theory.

A potential conclusion based on these facts, and particularly the latter one, is that the world cannot harbour a positive cosmological constant. In that case, the astronomical data must be interpreted to say that we are yet to settle into the true vacuum. Either we continue to roll slowly down a potential, realizing something like quintessence (see, e.g. [2]), or else we await a sharp phase transition in the future.

Regardless, during the epoch preceding descent to the true vacuum, the world is rapidly expanding as though there is a cosmological constant, and any inertial observer sees an apparent cosmological horizon. In recent years, evidence has arisen that the Bekenstein-Hawking entropy of event horizons can be extended to a more local statement regarding the entropy enclosed within, or encoded on, any closed surface [3]–[5]. This suggests that during the epoch of rapid expansion an inertial observer will appear to be in contact with a universe of finite entropy. We therefore return to the question of whether a rapidly expanding space like de Sitter actually has a finite entropy. If it is physically meaningful, the entropy of de Sitter, like any classical gravitational entropy, is a striking example of non-decoupling, where a large-scale feature of a gravitating system reflects microscopic details like the number of degrees of freedom available to the theory.

In this note, we present a picture of euclidean de Sitter space in string theory which allows us to relate de Sitter entropy to the number of degrees of freedom in a dual quantum mechanics. This leads to an argument that euclidean de Sitter spaces in divers dimensions may have dual descriptions in string theory in terms of certain quantum mechanical models.

2. Classic facts

It is worth our while to begin with a review of known facts about semiclassical de Sitter space.[1] De Sitter is a maximally symmetric solution to Einstein's equations with a positive cosmological constant Λ. Defining a length scale $\ell = \sqrt{1/\Lambda}$, D-dimensional de Sitter space (dS_D) has a metric:

$$ds^2 = -dt^2 + \ell^2 \cosh^2(t/\ell)\, d\Omega^2_{D-1}\,. \tag{2.1}$$

Equal time sections of this metric, which covers dS_D globally, are (D-1)-spheres (S^{D-1}). These spatial sections have no asymptotic region, and therefore there is no global notion of a conserved energy in de Sitter space [7].

Because of the exponential growth of the spatial sections at late times, an inertial observer in dS_D sees a cosmological horizon. The region of spacetime visible to such an observer can be described by a static metric:

$$ds^2 = -V(r)\, dt^2 + \frac{1}{V(r)}\, dr^2 + r^2\, d\Omega^2_{D-2}\,; \qquad V(r) = 1 - \frac{r^2}{\ell^2}\,. \tag{2.2}$$

Here polar coordinates have been constructed around an inertial observer placed at $r = 0$, which could be the north pole of the sphere in (2.1). The static patch displays an event horizon at $r = \ell$, and the inertial observer has access to phenomena within the region $r \leq \ell$. Essentially, de Sitter space can be viewed as a finite cavity

[1] For example, see [6].

surrounding the observer, with the horizon as its boundary. While the metric (2.1) does not have any globally defined timelike Killing vectors, the $\mathrm{SO}(D,1)$ isometry group does give rise to a timelike Killing vector inside the static patch (2.2). Abbott and Deser have shown that there is a perturbative positive energy theorem with respect to this Killing vector for excitations with support restricted to the static patch [8]. Therefore, the static patch of de Sitter space is perturbatively stable with respect to fluctuations that are accessible to its inertial observer.

Applying the Bekenstein-Hawking entropy formula to the cosmological event horizon yields an entropy

$$S = \frac{A}{4\,G_N} = \frac{(2\ell)^{D-2}\,\pi^{(D-1)/2}}{G_N\,\Gamma((D-1)/2)} \sim \frac{\Lambda^{-(D-2)/2}}{G_N}\,. \tag{2.3}$$

Gibbons and Hawking showed that the classical laws of horizon mechanics appearing in the physics of black holes apply equally to the de Sitter horizon, so that *every* inertial observer sees a world with horizons obeying the formal rules of thermodynamics [9] with area playing the role of entropy. It has been further argued that the entropy of empty de Sitter space bounds the total thermodynamic entropy of matter systems that can be placed within a horizon volume of any world with a positive cosmological constant Λ [4, 10, 11].

The analogy of static patch physics with thermodynamics is greatly strengthened by the remarkable observation that detectors held by any inertial observer in de Sitter space [9, 12], for example, one stationed at $r=0$ in the static patch (2.2) register a thermal bath with a temperature

$$T = \frac{1}{2\,\pi\,\ell}\,. \tag{2.4}$$

Every inertial observer in de Sitter space detects this temperature and a horizon with entropy (2.3) suggesting that these are properties of the spacetime, and not of the frames occupied by individual observers. To clarify the situation it is helpful to examine the thermodynamics of de Sitter space in semiclassical euclidean gravity.

2.1 Euclidean de Sitter space and thermodynamics

The euclidean continuation of the static patch (2.2) is obtained by rotating $t \to it$ and identifying $t \sim t + 2\pi\ell$, in order to obtain a non-singular space. Making the coordinate transformations $t = \ell\,\tau$ and $r = \ell\,\sin\theta$ gives the metric

$$ds^2 = \ell^2\,[d\theta^2 + \sin^2\theta\,d\Omega_{D-2}^2 + \cos^2\theta\,d\tau^2] = \ell^2\,d\Omega_D^2\,. \tag{2.5}$$

In other words, euclidean de Sitter (EdS) is a D-sphere with a round metric. The euclidean horizon is the set of fixed points of the time translation and is therefore a $(D-2)$-sphere of maximum size within EdS. The periodicity of euclidean time implies a temperature

$$T = \frac{1}{2\pi\ell}\,. \tag{2.6}$$

The euclidean continuation of global de Sitter time ($t \to it$, $t \sim t + 2\pi\ell$, $t = \ell\,\tau$) similarly yields

$$ds^2 = \ell^2 \left[d\tau^2 + \cos^2\tau \, d\Omega_{D-1}^2 \right], \tag{2.7}$$

which is the same as (2.5) in different coordinates; thus the round D-sphere is the unique euclidean continuation of lorentzian de Sitter space.[2]

The euclidean action with a positive cosmological constant is

$$I = \frac{1}{16\,\pi G_N} \int \sqrt{g}(R - 2\Lambda)\,. \tag{2.8}$$

Since EdS has no boundary there are no boundary terms in the euclidean action. As discussed by Gibbons and Hawking [13], the euclidean approach to gravitational thermodynamics requires us to evaluate the partition function with periodic boundary conditions in time:

$$Z = \mathrm{tr}\, e^{-\beta H} = \int \mathcal{D}\phi e^{-I(\phi)} \approx e^{-I_{cl}(\phi)}\,, \tag{2.9}$$

where ϕ includes all fields in the system and the last equality represents the semi-classical limit. In this limit, then,

$$\ln Z \approx I_{cl} \equiv \beta(E - T\,S)\,. \tag{2.10}$$

We can evaluate this on empty de Sitter space, using $R = 4\Lambda$ and the fact that $E = 0$ for a closed space like EdS (see, e.g. [7]). This gives

$$S \sim \frac{(2\ell)^{D-2}\,\pi^{(D-1)/2}}{G_N\,\Gamma((D-1)/2)} \sim \frac{\Lambda^{-(D-2)/2}}{G_N}\,. \tag{2.11}$$

Since both global de Sitter and the static patch are obtained as continuations of the same euclidean space, this suggests that de Sitter space is associated with an entropy $S \sim \ell^{D-2}/G_N$.

2.2 The meaning of entropy

The purpose of this note is to analyze whether and how the entropy of de Sitter space can be understood from string theory. But what does entropy mean in quantum gravity? Recapitulating controversies over black hole entropy, several interpretations are possible:

1. dS entropy is just a formal analogy and should not be interpreted a real thermodynamic quantity. We will set aside this skeptical position.

[2] Recall that global anti-de Sitter space (AdS) and the Poincaré patch both continue to the same euclidean manifold.

JHEP05(2001)043

2. dS entropy arises by quantizing degrees of freedom associated with a horizon. In dS_{2+1}, Carlip's horizon boundary conditions have yielded an account of de Sitter entropy [14, 15].[3] However, it is unclear at present how to extend this approach, relying on features of $2 + 1$ gravity, directly to higher dimensions. For some ideas in this direction see [17].

3. dS entropy arises from quantum entanglement with degrees of freedom that are hidden behind the cosmological horizon. If Newton's constant is wholly induced from matter fluctuations, black hole entropy can be understood in terms of entanglement [18]. In a brane-world scenario [19] this has been used to analyze de Sitter entropy.

4. dS entropy counts the number of initial conditions that can evolve into empty de Sitter space.

5. dS entropy counts the number of microscopic configurations that are macroscopically de Sitter. This is the approach that succeeded for a large class of extremal and near-extremal black holes in string theory [20].

6. dS entropy is the *finite* dimension of the Hilbert space describing the quantum gravity of de Sitter. This radical conclusion has been particularly advocated by Banks [21].

7. dS entropy counts the number of degrees of freedom of quantum gravity in de Sitter space.

Several of these philosophies have their roots in the principle of holography proposed by 't Hooft and Susskind [3].

Note that there is a difference between (6) and (7), since a single degree of freedom like a harmonic oscillator can have an infinite dimensional Hilbert space. Of course, at a fixed finite temperature, only a finite dimensional Hilbert space will be effectively accessible to a system with a finite number of degrees of freedom.

In any case, holographic arguments [3, 10, 11], [21]–[23] strongly suggest that de Sitter entropy is at least meaningful as a bound on the functionally accessible entropy of matter systems within one horizon volume of a world with a cosmological constant. We might suppose that this quantity is relevant to an inertial observer's effective description of the world within a static patch, and that this is the right way to think about de Sitter physics since there is a positive energy theorem for the relevant fluctuations [8]. To make sense of physics in global coordinates, it has been argued that different static patches should be considered gauge copies of each other so that dS entropy measures a property of the entire universe [21, 23]. In our approach, we will evade this interpretational issue by simply discussing the entropy of de Sitter space in the *euclidean* framework.

[3] A different approach has appeared recently in [16].

3. Seeking de Sitter space

No go: any attempt to realize de Sitter space within string theory must contend with several no go theorems in the literature. These include one stating that de Sitter compactifications of conventional supergravity cannot be found under the following conditions [24]: (*a*) The action does not contain higher derivative terms, (*b*) The scalars have a potential $V(\phi) \leq 0$, (*c*) The form fields have positive kinetic terms, (*d*) The Newton constant on de Sitter is finite, namely gravitons interact with a finite strength. The low energy gravities arising from string theory certainly have higher derivative corrections. Likewise, not all scalar potentials arising in string theory are strictly non-negative. However, we will not explore these avenues in this paper. There is also a theorem stating that 11d supergravity does not admit a cosmological constant [25], and therefore does not give rise to the 11d de Sitter space. Finally, attempts to construct a maximal ($\mathcal{N}=8$) cosmological supergravity theory in four dimensions by starting with an Einstein term and a positive cosmological constant [26] revealed that de Sitter superalgebras do exist, but result in a lagrangian with two closely related unpalatable features: (*a*) the R-symmetry group $\mathrm{SO}(6,2)$ is non-compact, and therefore (*b*) the gauge field has some ghostlike components with the wrong sign kinetic term. Another curious feature of the de Sitter superalgebra is that the supercharges square to zero [26],

$$\sum \{Q, Q^*\} = 0\,, \tag{3.1}$$

instead of anticommuting to the hamiltonian, $\sum\{Q, Q^*\} = H$, as usual. Note that the absence of H on the right hand side of (3.1) implies that a positive energy theorem cannot be proved for de Sitter space by appealing to the supersymmetry algebra. What is more, the de Sitter superalgebras have no non-trivial representation on a positive Hilbert space, which certainly precludes any naive attempt to construct a Fock space of multi-particle states. These facts seem to suggest a possible connection with topological field theory.[4]

Go?: hull and Warner showed that de Sitter spaces can arise from compactification of supergravity on hyperbolic manifolds. Problems with wrong sign kinetic terms were avoided in this approach by squashing the hyperbolic space to remove non-compact isometries [28]. There have been attempts to escape these no go theorems by changing the rul1es of the theories in which they were derived. Hull [27] and Hull and Khuri [29] have discussed type-II* strings and M* theory which can arise from timelike T-dualities of the conventional systems. These theories can have a variety of different spacetime signatures and admit euclidean brane solutions whose near-horizon limits realize de Sitter space. In another approach, Chamblin and Lambert [30] have discussed a massive form of the 11d supergravity equations that admit

[4] A relation between de Sitter and topological field theories has been explored by Hull [27]. The $\mathrm{SO}(6,2)$ de Sitter gravity is also presented in these papers.

de Sitter solutions. It is unclear at present whether these theories can be made stable and quantum mechanically consistent.

Go around: we will evade the no go theorems reviewed above by not requiring that dimensional reduction of the fundamental theory on a compact manifold yields de Sitter space. Rather, we are inspired by the result of [26] that a maximal supergravity on dS_4 would have an $SO(6,2)$ R-symmetry group. This is suggestive because that $SO(6,2)$ is the isometry group of AdS_7. M-theory admits $S^4 \times AdS_7$ solutions in which the sphere is supported by a four-form flux passing through it. In these compactifications, the isometry group of the S^4 acts as the $SO(5)$ R-symmetry group of supergravity on AdS_7. Turning things around we might expect that a theory on S^4 obtained by "compactification" of M-theory on AdS_7 should have an $SO(6,2)$ R-symmetry arising from the AdS isometry group. Now recall that S^4 is exactly the same manifold as 4d, euclidean de Sitter space, EdS_4, i.e., $S^4 \times AdS_7 \equiv EdS_4 \times AdS_7$. We therefore suggest that:

> *The $S^D \times AdS_{k+1}$ solutions of M-theory should be regarded as $EdS_D \times AdS_{k+1}$ compactifications. In this picture, the cosmological constant is implemented by a D-form flux passing through the de Sitter world.*

We will examine this proposal in the four basic $EdS_D \times AdS_{k+1}$ compactifications of M-theory, which have $(D, k+1) = (5,5), (3,3), (4,7), (7,4)$.

At first sight there are two problems with this proposal. First of all, in usual compactifications, the Newton constant on the noncompact factor is given by the higher dimensional G_N divided by the volume of the internal space. AdS has an infinite volume and this threatens to give an effective vanishing Newton constant on the EdS side. Secondly, lorentzian time is in the wrong place — it is in the AdS factor.[5] We will address these issues in turn.

3.1 Localization on AdS

We are interested in the physics of fluctuations of EdS and not of AdS. Therefore we should isolate a sector of the dynamics of $EdS_D \times AdS_{k+1}$ which describes fluctuations of the sphere. To do this we focus on supergravity modes that are minimally excited on the anti-de Sitter factor. The metric of AdS creates an effective potential well that localizes states of finite energy, although the space is non-compact. This is in marked contrast to flat space. This feature implies that the modes of interest to us effectively explore a finite volume of AdS.

To illustrate these points, it is helpful to examine the scalar wave equation on $EdS_D \times AdS_{k+1}$. Placing a mode in a spherical harmonic on the EdS factor yields a

[5] Possible compactifications on a euclidean hyperbolic space to a Lorentz-signature de Sitter have been discussed by Hull [27]. This tempting "double Wick rotation" of the AdS backgrounds requires Ramond-Ramond fields with the wrong-sign kinetic terms, and therefore belongs to the realm of M*-theory.

JHEP05(2001)043

massive wave equation $(\nabla^2 + m^2)\phi = 0$, where m^2 is the eigenvalue of the laplacian on the sphere. Working with a global AdS metric

$$ds^2 = L^2[-\sec^2\rho\, dt^2 + \sec^2\rho\, d\rho^2 + \tan^2\rho\, d\Omega_{k-1}^2]\,, \tag{3.2}$$

the normalizable mode solutions to the wave equation on AdS are:

$$\phi_+ \propto e^{-i\omega t} Y_{lp}(\cos\rho)^{2h_+}(\sin\rho)^l_2 F_1\left(h_+ + \frac{l+\omega}{2}, h_+ + \frac{l-\omega}{2}, l + k/2; \sin^2\rho\right), \tag{3.3}$$

$$2h_+ = \frac{k}{2} + \frac{\sqrt{k^2 + 4m^2L^2}}{2}\,, \tag{3.4}$$

$$\omega L = 2h_+ + l + 2n\,. \tag{3.5}$$

Here Y_{lp} is a spherical harmonic on the AdS factor of the space, ${}_2F_1$ is a hypergeometric function [31] and L is the AdS radius. (Recall that in the AdS $\times$ EdS compactifications of string theory L is equal to ℓ, the EdS scale, up to factors of two.) In particular, ground states, with $n = l = 0$, have the form:

$$\phi^+ \propto e^{-i\omega t}\,(\cos\rho)^{2h_+} \tag{3.6}$$

and are therefore localized within one AdS radius. It is easy to check that as l increases, the modes (3.3) are pushed rapidly outward by the angular momentum barrier, and represent ripples of AdS, and not fluctuations of EdS. On the other hand, as n increases, the wavefunction remains substantially localized inside the AdS scale. Higher harmonics on EdS increase the effective mass m^2 and hence h_+, localizing the $l = 0$ modes still further. Although we have illustrated these point by studying scalar fields on AdS, similar behaviour occurs for higher spin fields. This analysis suggests that all the physics of EdS is already contained in modes that can be localized within one AdS radius.

One might prefer an alternative point of view in which we simply impose a cutoff at one AdS radius. In the CFT dual to AdS space, such a cutoff should correspond to a reduction to a matrix theory [32, 33]. We will later consider the consequences of enlarging the cutoff. However, we cannot decrease it without cutting off part of the wavefunctions of ground state modes on AdS.

Among the modes that we have admitted are the giant gravitons that expand inside EdS. These states, with $l = n = 0$, are subject to a bound on h_+ [34], and expand into co-dimension two spheres on EdS. Interestingly, the largest giant graviton fills out the euclidean de Sitter horizon.

3.2 "Compactifying" on AdS

We have argued that a finite spatial volume of AdS is accessible during interactions between the modes of interest for EdS physics. This does not yet imply a finite Newton constant on EdS, as we are still left with the non-compact time dimension in the AdS factor. In order to complete the effective compactification on AdS we will now make time in (3.2) periodic.

Supergravity modes on $\mathrm{AdS}_{k+1} \times \mathrm{EdS}_D$ are known to have quantized frequencies in units of the AdS scale.[6] In essence, this happens because, despite its non-compactness, the AdS geometry creates an effective potential well for the supergravity fluctuations. Therefore, at the level of supergravity, we can compactify time with a period $\sim L$.

Having made time periodic, we can also make it euclidean at the same period. There are no normalizable mode solutions in euclidean time, and the frequencies of non-normalizable modes are not quantized. The compactification of time projects out many of these modes, and only a quantized set of frequencies survives. Earlier we argued that the physics of EdS is already contained within the normalizable modes which are localized within one AdS radius. The analogous statement in euclidean signature is that the non-normalizable modes of interest show pure scaling behaviour outside an AdS radius.

Estimating G_N: we have argued that all the physics of EdS is already contained in the physics of excitations within one AdS radius L. The interaction strengths of such modes are governed by the overlaps of their localized wavefunctions on the AdS side. Since these wavefunctions are effectively contained in finite spatial volume $\sim L^k$, their overlap can be estimated by cutting off the AdS space at one AdS radius and including the volume factor L^k in the effective interaction strength of harmonic modes on the sphere. Moreover, the compactification of the AdS time at the scale L implies that the effective volume of the AdS space-time accessible to the modes of interest is finite, and scales as L^{k+1}. Therefore, the effective Newton constant for the excitations of the EdS_D factor is finite, and scales as

$$G_D \sim \frac{G_N}{L^{k+1}} \sim \frac{G_N}{\ell^{k+1}} \,. \tag{3.7}$$

Here, and in future equations, we will only keep track of the scaling of quantities, and will not attempt to determine numerical factors.

4. De Sitter entropy

Using our estimate of G_N we can examine the entropy of the EdS factors in the EdS $\times$ AdS compactifications of string theory.

EdS $_5 \times$ AdS$_5$: the geometric area of the euclidean horizon of EdS_5 in units of the effective Newton constant G_5 gives the entropy:

$$S_5 \sim \frac{\ell^3}{G_5} \,. \tag{4.1}$$

[6]For references, see the review [35].

We have argued that because the modes of interest for the study of EdS physics are effectively localized on AdS,

$$G_5 \sim \frac{G_{10}}{\ell^5}\,. \tag{4.2}$$

The $\mathrm{EdS}_5 \times \mathrm{AdS}_5$ compactification of IIb string theory is dual to the superconformal Yang-Mills theory of the $\mathrm{SU}(N)$ gauge group, defined on the 4d AdS_5 boundary. The EdS radius is given in terms of the string length and N as [36]

$$\ell \sim l_s (g_s N)^{1/4}\,. \tag{4.3}$$

In these variables the entropy is given by

$$S_5 \sim N^2\,. \tag{4.4}$$

All factors of l_s had to cancel for dimensional reasons, but it is significant that the string coupling g_s drops out. Since the matrices of the CFT are $N \times N$, the quantum mechanics obtained by dimensionally reducing it to a point will have

$$N_{\mathrm{d.o.f.}} \sim N^2 \tag{4.5}$$

degrees of freedom. In short, the gravitational entropy of EdS_5 (4.4) matches the number of degrees of freedom (4.5) in the matrix mechanics. This is in accord with the expectation that the region in the deep interior of AdS, which we have argued contains the modes relevant for EdS physics, has a dual description in a matrix model [33, 32].

EdS $_7$ × AdS$_4$: in this case, the gravitational entropy is $S_7 \sim \ell^5/G_7$, in terms of ℓ, the radius of EdS_7 and G_7, the effective 7d Newton constant. Following earlier reasoning, $G_7 \sim l_p^9\, \ell^{-4}$, where l_p is the 11-dimensional Planck length. The $\mathrm{EdS}_7 \times \mathrm{AdS}_4$ compactification of M-theory is dual to the strong coupling limit of the $\mathrm{SU}(N)$ SYM in $2+1$ dimensions. This is a 3d CFT with SO(8) R-symmetry arising from N coincident M2-branes. According to the AdS/CFT dictionary $\ell \sim N^{1/6} l_p$ [36]. Assembling these facts, the gravitational entropy of EdS_7 is $S_7 \sim N^{3/2}$. Although the dual CFT is strongly coupled, the Bekenstein-Hawking entropy of non-extremal 2-branes suggests that there are $O(N^{3/2})$ independent degrees of freedom in it [37]. Presumably, dimensionally reducing to a point gives a quantum mechanics with $O(N^{3/2})$ degrees of freedom, which matches the scaling of the EdS_7 entropy.

EdS $_4$ × AdS$_7$: the gravitational entropy of EdS_4 is $S_4 \sim \ell^2/G_4$. As discussed, $G_4 \sim l_p^9\, \ell^{-7}$. The $\mathrm{EdS}_4 \times \mathrm{AdS}_7$ compactification of M-theory is dual to the strongly coupled $(2,0)$ CFT in 6 dimensions. This is the worldvolume theory of N coincident M5-branes. The AdS/CFT dictionary states that $\ell \sim N^{1/3} l_p$. These relations yield an EdS_4 entropy $S_4 \sim N^3$. The Bekenstein-Hawking entropy of the non-extremal M5-brane system suggests that there are $O(N^3)$ degrees of freedom in the $(2,0)$ theory [37]. We would expect that dimensionally reducing to a point yields a quantum mechanics with N^3 degrees of freedom, matching the EdS_4 entropy.

$\mathrm{EdS}_3 \times \mathrm{AdS}_3$: finally, consider the $\mathrm{EdS}_3 \times \mathrm{AdS}_3 \times \mathcal{M}$ compactifications of IIB string theory, with $\mathcal{M} = K3$ (T^4). The gravitational entropy is $S_3 \sim \ell/G_3$, where ℓ is the radius of EdS_3 and $G_3 \sim g_s^2\, l_s^4\, \ell^{-3}$. Furthermore, $\ell^2 \sim l_6^2 \sqrt{N}$ where l_6 is the 6d Planck length resulting from the compactification on $K3$ (T^4). Putting these facts together, the gravitional entropy is $S_5 \sim N$. The dual theory is the $(4,4)$ symmetric product sigma model on the target $(K3)^N/S_N$ $((T^4)^N/S_N)$. The central charge of this theory is $c \sim N$, and so we expect that the quantum mechanical reduction has $\sim N$ degrees of freedom. Again, this coincides with the EdS_3 entropy.

We have argued that in a variety of examples, the number of degrees of freedom of a quantum mechnical model derived from the AdS/CFT correspondence matches the entropy of euclidean de Sitter space.

This picture makes one wonder whether there is a precise dual to euclidean de Sitter space in terms of a matrix quantum mechanics. We have suggested that the physics of euclidean de Sitter can be understood in terms of the lowest modes of fields which are localized within one AdS radius. In the dual CFT, this sector is described by a quantum mechanical theory in which the spatial dynamics has been completely suppressed. There are two ways of seeing this. First of all, the duals to the bulk $l = 0$ states are created by the lowest Fourier modes of CFT operators. What is more, the UV/IR connection in the AdS/CFT correspondence, and the holographic renormalization group [38, 39], teach us that phenomena localized deep in the AdS interior should be described by the long wavelength limit of the CFT. In particular, we expect that the physics of EdS that we are trying to isolate is contained within the quantum mechanical theory obtained by dimensionally reducing the dual CFT to a point [33].

One can effectively view our calculation in this section as cutting off AdS space at a radius $r \sim \ell$ to get a finite Newton constant on EdS. Instead of keeping the physics inside one AdS radius, we could have chosen a different cutoff $\varepsilon\ell$, effectively localizing inside a bigger portion of AdS. The holographic argument of Susskind and Witten [32] implies that the number of CFT degrees of freedom required to describe this region is then rescaled by ε^k. This change of the cutoff also rescales the effective G_D on EdS by $1/\varepsilon^k$, and therefore the Bekenstein-Hawking entropy is multiplied by ε^k. Therefore, the relation between the entropy of EdS and the number of degrees of freedom in the quantum mechanical dual scales correctly with changes in the cutoff.

5. Discussion

In summary, we have argued that euclidean de Sitter space makes an appearance in string theory as the sphere in the AdS $\times$ S compactifications. The dynamics of EdS is carried by modes that are localized on AdS, yielding a finite effective Newton constant on the EdS factor. The resulting de Sitter entropy, measured as the area of the horizon in units of this effective G_N, scales as the number of degrees of freedom in the corresponding dual.

JHEP05(2001)043

Our construction does not provide a decoupled theory of de Sitter gravity. Rather, the ambient AdS space acts like a regulator for the de Sitter theory, in much the same way that the massive states of strings regulate a theory of gravity that would otherwise be ill-defined.

The effective cutoff in AdS space certainly breaks conformal invariance. Since de Sitter space has a finite temperature, we also expect that supersymmetry should also be broken by the choice of the spin structure around the AdS time. An alternative way of making the Newton constant on EdS finite would be to orbifold AdS by a sufficiently large discrete symmetry group to get a space of finite volume. Precedents in this regard include [40].

It remains a challenge to continue our picture to lorentzian de Sitter space. Since we started with lorentzian time in the AdS factor, we must either move it to de Sitter or deal with a theory with two times. The second option arises if we rotate the EdS time back to lorentzian signature while leaving AdS untouched. These geometries appear as formal solutions of M* and type-II* theories [27, 29]. If we eliminate the lorentzian AdS time by Wick rotation, the Ramond-Ramond fields pick up wrong-sign kinetic terms [27, 29]. Clearly, there are questions about the stability and the quantum mechanical definition of these theories. Still, one is left with a sense that de Sitter is asking for two times. (In this regard, recall the non-compact R-symmetry groups in de Sitter superalgebras [26].)

An alternative approach is to attempt to make sense of the one lorentzian time that is already present on the AdS side, perhaps by "grafting" it onto the EdS time. Consider choosing a new generator of lorentzian time translations $H' = H + J$, where H is the hamiltonian on AdS and J is the EdS rotation generator corresponding to euclidean time translations. This procedure is somewhat reminiscent of topological twisting, since it mixes the original hamiltonian with an R-symmetry generator.

If we can make sense of our picture in lorentzian signature, it would be natural to attempt an interpretation of de Sitter entropy in terms of a counting of states. Which states should be counted? There is a natural bound on the energy of excitations that can be localized within a de Sitter volume. This bound is given by the mass of the largest black hole that can be placed within a horizon volume. The mass of the D-dimensional extremal Schwarschild-de Sitter solution, known as the Nariai black hole [41], is given by

$$M \sim \frac{\ell^{D-3}}{G_D}\,. \tag{5.1}$$

In the CFT variables of our scenario ℓ^{D-2}/G_D scales precisely as the number of degrees of freedom, or central charge c, of the dual theory. Therefore, the Nariai bound for the bulk energy translates into a bound on dimensionless CFT energies of the form $E \equiv \ell M \leq c$. This picture would suggest that the entropy of de Sitter space should be interpreted in terms of the finite dimension of the Hilbert space of states satisfying the Nariai bound.

JHEP05(2001)043

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JHEP05(2001)043